Homework 1: Derivative, Random Numbers

Exercise 8.1: Most Probable velocity:

The distribution of velocity of molecules in a gas is given by $f(v) = v^5 e^{-v^2}$. Find the most probable velocity. Apply your knowledge of calculating derivative numerically by central difference method. Use Δv = 0.05. Compare the result with exact value.

Exercise 8.2: Position, speed, acceleration:

The following table gives the position of a particle in one dimension as a function of time. Find its speed at $t = 0.1, 0.3, 0.5, ..., 1.9$ s. Find if its position has a maximum or a minimum during this interval. At which time is the particle located at the maxima/minima? Find its acceleration at that point to figure out if it is a maximum or a minimum.

t (s)	x(t) (cm)
0.0	0.0
0.2	0.16
0.4	0.27
0.6	0.33
0.8	0.36
1.0	0.37
1.2	0.36
1.4	0.35
1.6	0.32
1.8	0.30
2.0	0.27

Exercise 8.3: Random Walk:

This is an extension of the random walk problem given in class. A particle is confined to a square grid or lattice $L \times L$ squares on a side, so that its position can be represented by two integers $i, j = 0 \ldots L-1$. It starts in the middle of the grid. On each step of the simulation, choose a random direction—up, down, left, or right—and move the particle one step in that direction. This process is called a random walk. The particle is not allowed to move outside the limits of the lattice—if it tries to do so, choose a new random direction to move in.

Write a program to perform $N = 10^4$ steps of this process on a lattice with $L = 101$. Repeat the same for 50 particles. Write a table in an output file called "distancevsstep.out" containing two columns. Column one is the time step (i.e., it runs from 1 to 10^4) and column two is the average distance ($\bar{x}$) of all 50 particles from the center after each time step. Check if $\bar{x} \propto \sqrt{N}$.

Exercise 8.4: Rutherford Scattering

a) Starting with two numbers z and θ drawn from two uniform random distributions between 0 and 1, and 0 and 2π, respectively, construct two random numbers (x,y) in a Gaussian distribution by the following:

$$r = \sqrt{-2\sigma^2 ln(1-z)}$$

$$x = rcos\theta$$

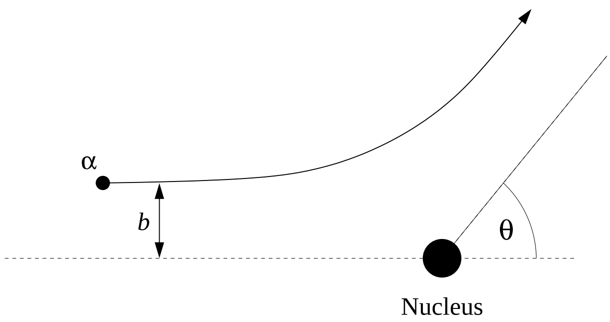
$$y = rsin\theta,$$

where σ is a constant that you may choose. Check that the distribution of a large number of x and y are indeed Gaussian.

b) Consider a beam of α particles with energy 7.7 MeV that has a Gaussian profile in both x and y axes with standard deviation $\sigma = a_0/100$, where a_0 is the Bohr radius. The beam is fired directly at a gold atom. Simulate the scattering of 10^6 α particles and calculate the fraction that "bounces back," i.e., scatters through an angle (θ) greater than 90°. The scattering angle θ is given by

$$tan\frac{1}{2}\theta = \frac{Ze^2}{2\pi\epsilon_0 Eb},$$

where Z is the atomic no. of the nucleus, e is the electron charge, ϵ_0 is the permittivity of free space, E is the kinetic energy of the incident



α particle, and b is the impact parameter. For $\theta = 90°$,

$$b = \frac{Ze^2}{2\pi\epsilon_0 E}.$$

If b is less than the above value the particle bounces back.

You may use the following values:

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Z = 79
e = 1.602e-19
E = 7.7e6*e
epsilon0 = 8.854e-12
a0 = 5.292e-11
sigma = a0/100
N = 1000000
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