Lab Exercise 1: Derivative, Random Numbers

Exercise 1.1: Derivative of a function: Find $\frac{dy}{dx}$ for $y(x) = x^2 e^{-x}$ at $x = 1$ using both forward difference (FD) and central difference (CD) methods for h=0.02,0.04,0.06,0.08,0.10. Make a table with the following 5 columns: $\frac{dy}{dx}|_{Exact}$, $\frac{dy}{dx}|_{FD}$, error (FD), $\frac{dy}{dx}|_{CD}$, error (CD), and five rows corresponding to the five values of h. Plot error (FD)/h vs h and error (CD)/h^2 vs h on the same graph. Make a pdf output of the graph. Error is defined as the difference between the exact and numerical value.

Exercise 1.2: Derivative from Discrete Data Points: The following table gives the position of a particle in one dimension as a function of time. Find its speed at $t = 0.1, 0.3, 0.5, ..., 1.9$ s. Find if its position has a maximum or a minimum during this interval. At approximately which time is the particle located at the maxima/minima? Find its acceleration at that point to figure out if it is a maximum or a minimum.

t (s)	x(t) (cm)
0.0	0.0
0.2	0.16
0.4	0.27
0.6	0.33
0.8	0.36
1.0	0.37
1.2	0.36
1.4	0.35
1.6	0.32
1.8	0.30
2.0	0.27

Exercise 1.3: The semi-empirical mass formula In nuclear physics, the semi-empirical mass formula is a formula for calculating the approximate nuclear binding energy B of an atomic nucleus with atomic number Z and mass number A:

$$B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \frac{a_5}{A^{1/2}},$$

where, in units of millions of electron volts, the constants are $a_1 = 15.8$, $a_2 = 18.3$, $a_3 = 0.714$, $a_4 = 23.2$, and

$$a_5 = \begin{cases} 0 & \text{if } A \text{ is odd,} \\ 12.0 & \text{if } A \text{ and } Z \text{ are both even,} \\ -12.0 & \text{if } A \text{ is even and } Z \text{ is odd.} \end{cases}$$

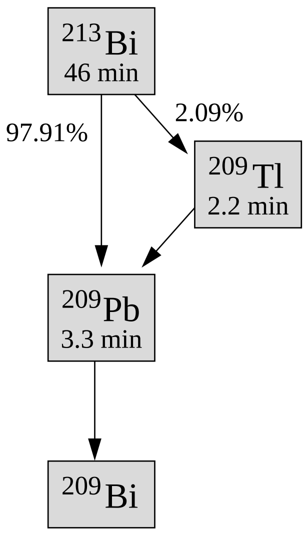
a) Write a program that takes as its input the values of A and Z, and prints out the binding energy for the corresponding atom. Use your program to find the binding energy of an atom with $A = 58$ and $Z = 28$. (Hint: The correct answer is around 490 MeV.)

b) Modify your program to print out not the total binding energy B, but the binding energy per nucleon, which is B/A.

c) Now modify your program so that it takes as input just a single value of the atomic number Z and then goes through all values of A from $A = Z$ to $A = 3Z$, to find the one that has the largest binding energy per nucleon. This is the most stable nucleus with the given atomic number. Have your program print out the value of A for this most stable nucleus and the value of the binding energy per nucleon.

d) Modify your program again so that, instead of taking Z as input, it runs through all values of Z from 1 to 100 and prints out the most stable value of A for each one. At what value of Z does the maximum binding energy per nucleon occur? (The true answer, in real life, is $Z = 28$, which is nickel. You should find that the semi-empirical mass formula gets the answer roughly right, but not exactly.)

Exercise 1.4: Radioactive decay chain The isotope ^{213}Bi decays to stable ^{209}Bi via one of two different routes, with probabilities and half-lives thus:



Starting with a sample consisting of 10 000 atoms of ^{213}Bi, simulate the decay of the atoms as in Example 10.1 by dividing time into slices of length $\delta t = 1$ s each and on each step doing the following:

a) For each atom of ^{209}Pb in turn, decide at random, with the appropriate probability, whether it decays or not. (The probability can be calculated from Eq. (10.3).) Count the total number that decay, subtract it from the number of ^{209}Pb atoms, and add it to the number of ^{209}Bi atoms.

b) Now do the same for ^{209}Tl, except that decaying atoms are subtracted from the total for ^{209}Tl and added to the total for ^{209}Pb.

c) For ^{213}Bi the situation is more complicated: when a ^{213}Bi atom decays you have to decide at random with the appropriate probability the route by which it decays. Count the numbers that decay by each route and add and subtract accordingly.

Note that you have to work up the chain from the bottom like this, not down from the top, to avoid inadvertently making the same atom decay twice on a single step.

Keep track of the number of atoms of each of the four isotopes at all times for 20 000 seconds and make a single graph showing the four numbers as a function of time on the same axes.